V2G3C3

The question book

Creative problem solving

Acknowledgements

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The question book

Every species on this earth faces problem, big or small and finds probable solution(s) for it through various ways as per their capabilities and limitations. Problem solving is a daily activity for most of us, whether we realize it or not. Whenever there is a gap between where you are now and where you want to be and you don't know how to cross that gap, you have a problem1 . For example, if you want to go to your friends place and there are three paths to take and you don't know how to find the best path to take, that is a problem and when you figure out the best path after looking into various aspects that's problem solving.

Learning and understanding the process of problem solving and recognizing patterns in problems is a lifelong activity and a skill that can be applied both in personal and
professional lives. This is one of the most essential 21 st century skills for anyone to learn.

The question book in your hand will take you through a journey where you will find various kinds of problems and discussions on them to find the probable solutions after looking into pros and cons of various aspects of it. It will also give you an opportunity to identify the problems in your daily life for which you will find best solutions yourself.

- Ankit and Ankita

Hays, 1980

Central Board of Secondary Education
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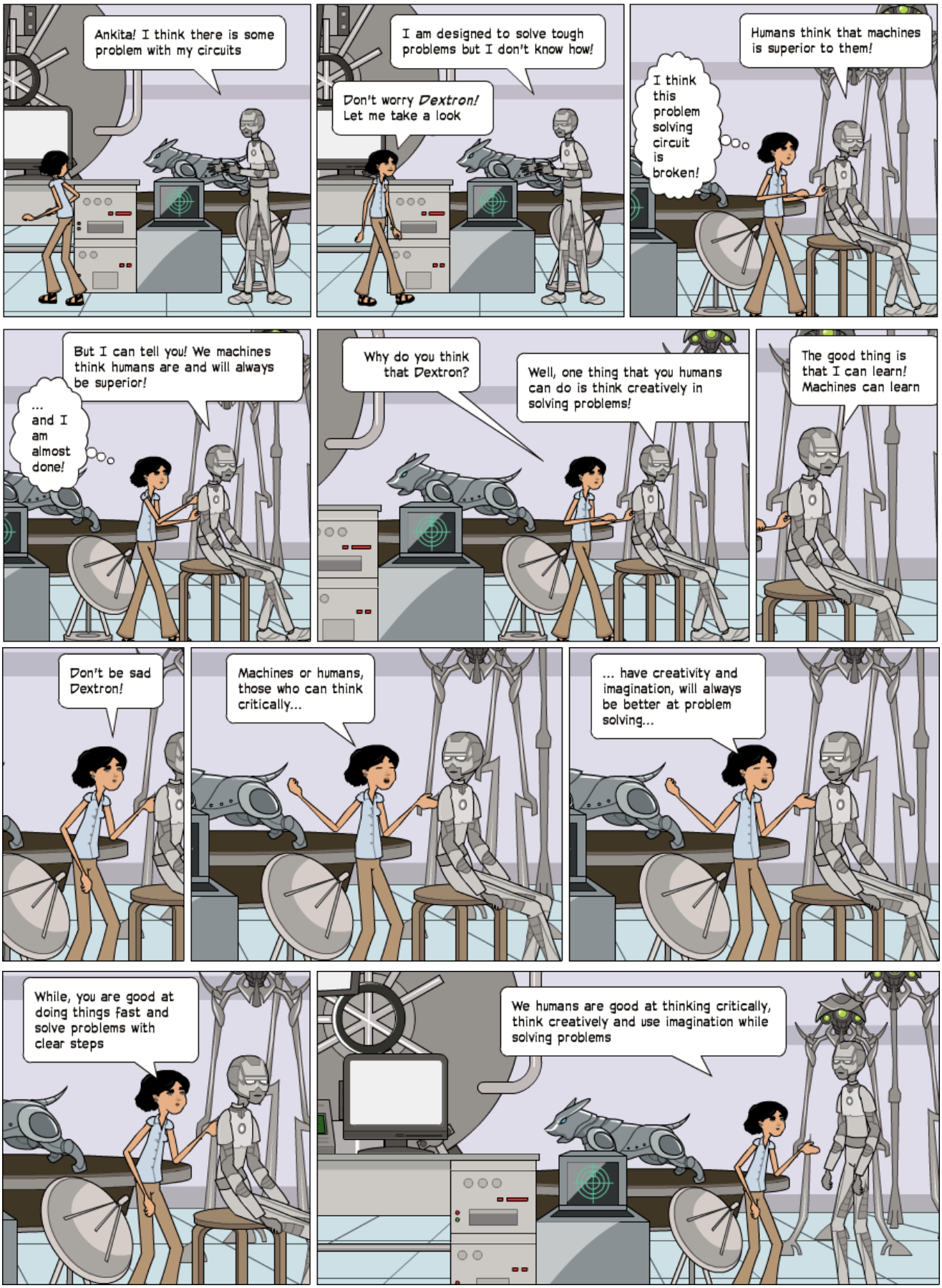
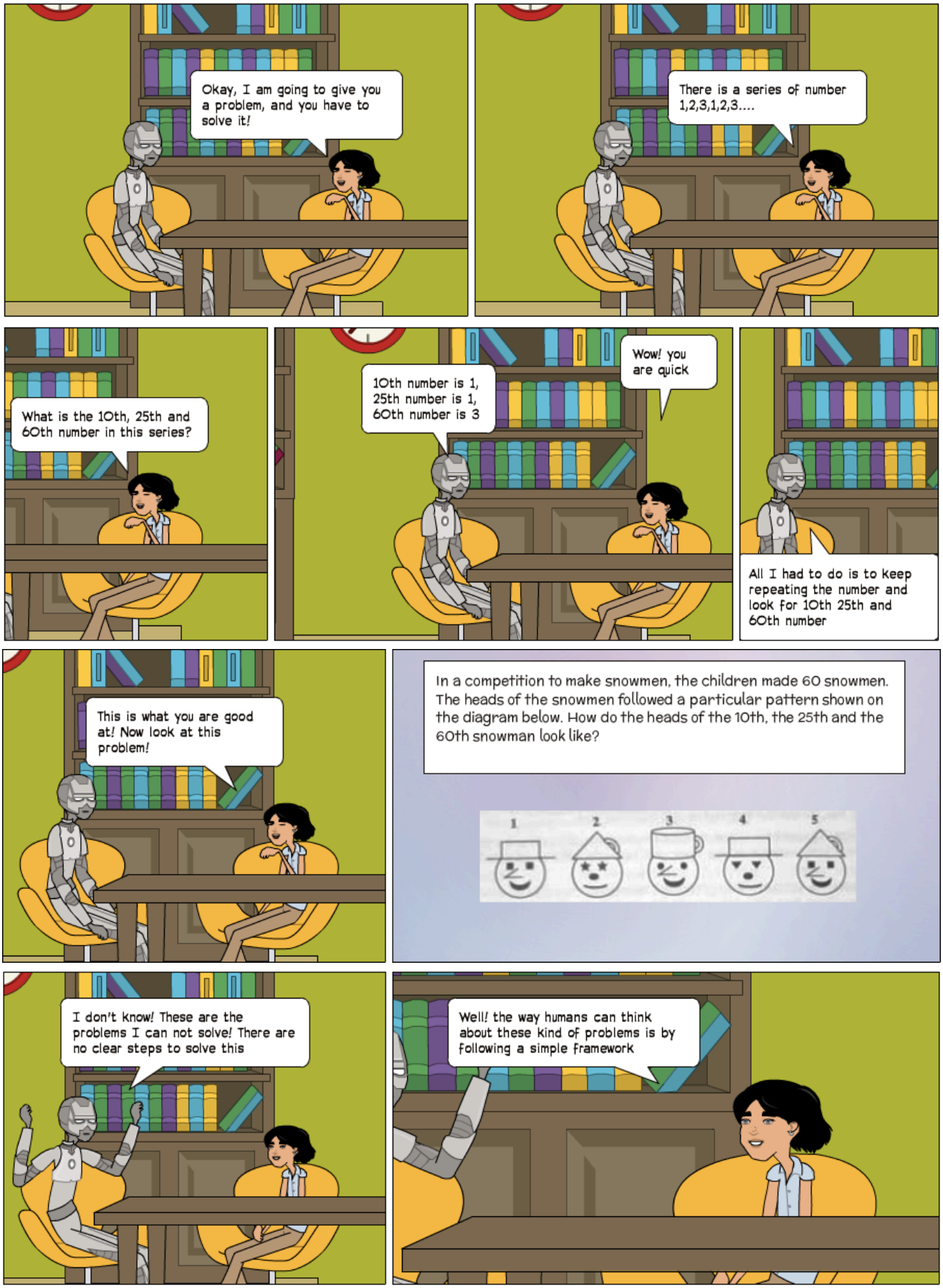
The question book


Ankita! I think there is some problem with my circuits
I am designed to solve tough problems but I don't know how!
Don't worry Dextron! Let me take a look
Humans think that machines is superior to them!
I think this problem solving circuit is broken!
But I can tell you! We machines think humans are and will always be superior!
... and I am almost done!
Why do you think that Dextron?
Well, one thing that you humans can do is think creatively in solving problems!
The good thing is that I can learn! Machines can learn
Don't be sad Dextron!
Machines or humans, those who can think critically...
... have creativity and imagination, will always be better at problem solving...
While, you are good at doing things fast and solve problems with clear steps
We humans are good at thinking critically, think creatively and use imagination while solving problems


So I am going to teach you how to solve problems!
Why do we always come to library?
Because, if you can read, you can learn, if you can learn you think, if you can think you can be the best!

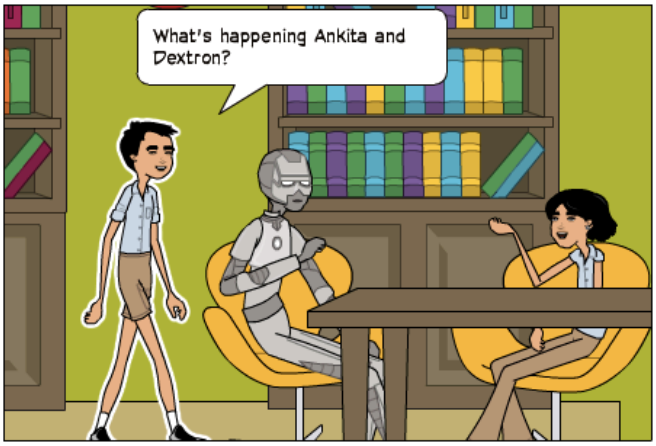
Step by Step

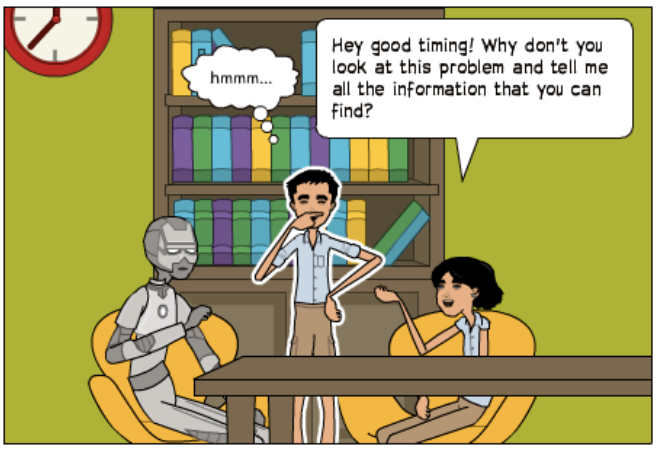

Okay, I am going to give you a problem, and you have to solve it!
There is a series of number 1,2,3,1,2,3....
What is the 10th, 25th and 60th number in this series?
10th number is 1, 25th number is 1, 60th number is 3
Wow! you are quick
All I had to do is to keep repeating the number and look for 10th 25th and 60th number
This is what you are good at! Now look at this problem!
In a competition to make snowmen, the children made 60 snowmen. The heads of the snowmen followed a particular pattern shown on the diagram below. How do the heads of the 10th, the 25th and the 60th snowman look like?
1
2
3
4
5
I don't know! These are the problems I can not solve! There are no clear steps to solve this
Well! the way humans can think about these kind of problems is by following a simple framework

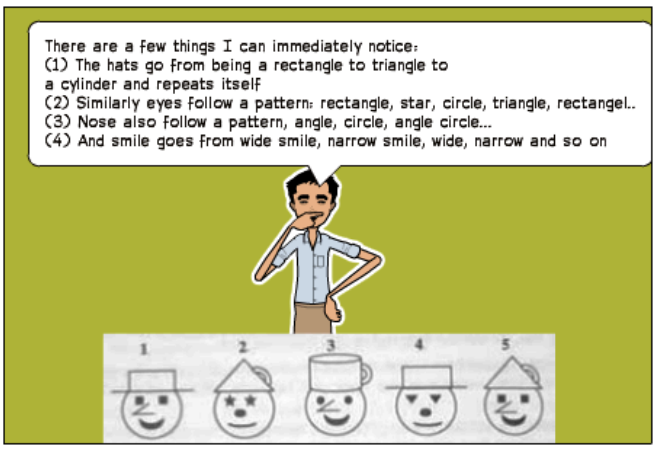
Step 1: identifying which pieces of data are relevant when faced with a mass of data, most of which is irrelevant

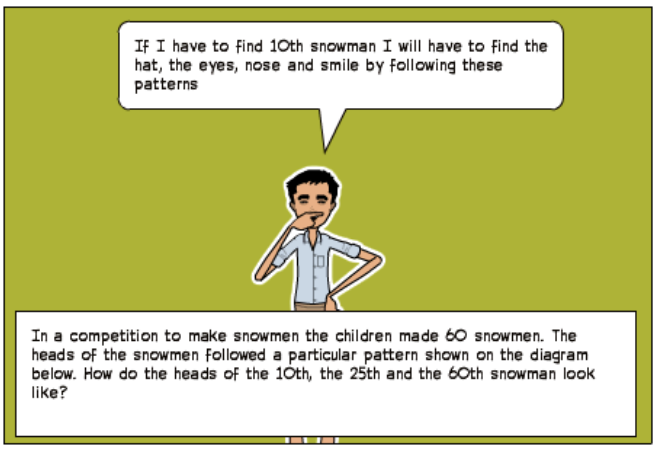
Step 2: combining pieces of information that may not appear to be related to give new information

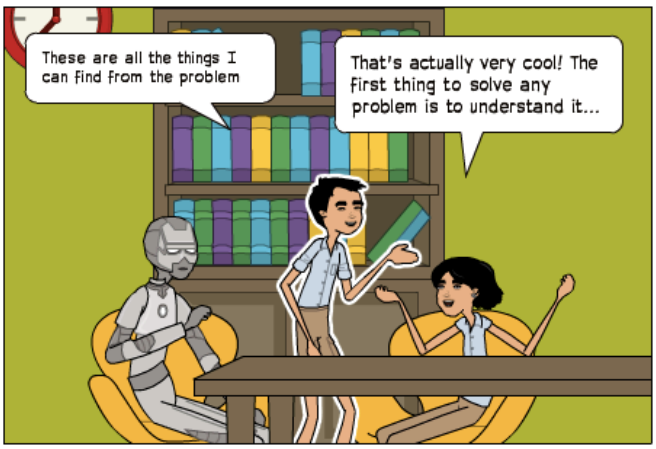
Step 3: relating one set of information to another in a different form – this involves using experience: relating new problems to ones we have previously solved.

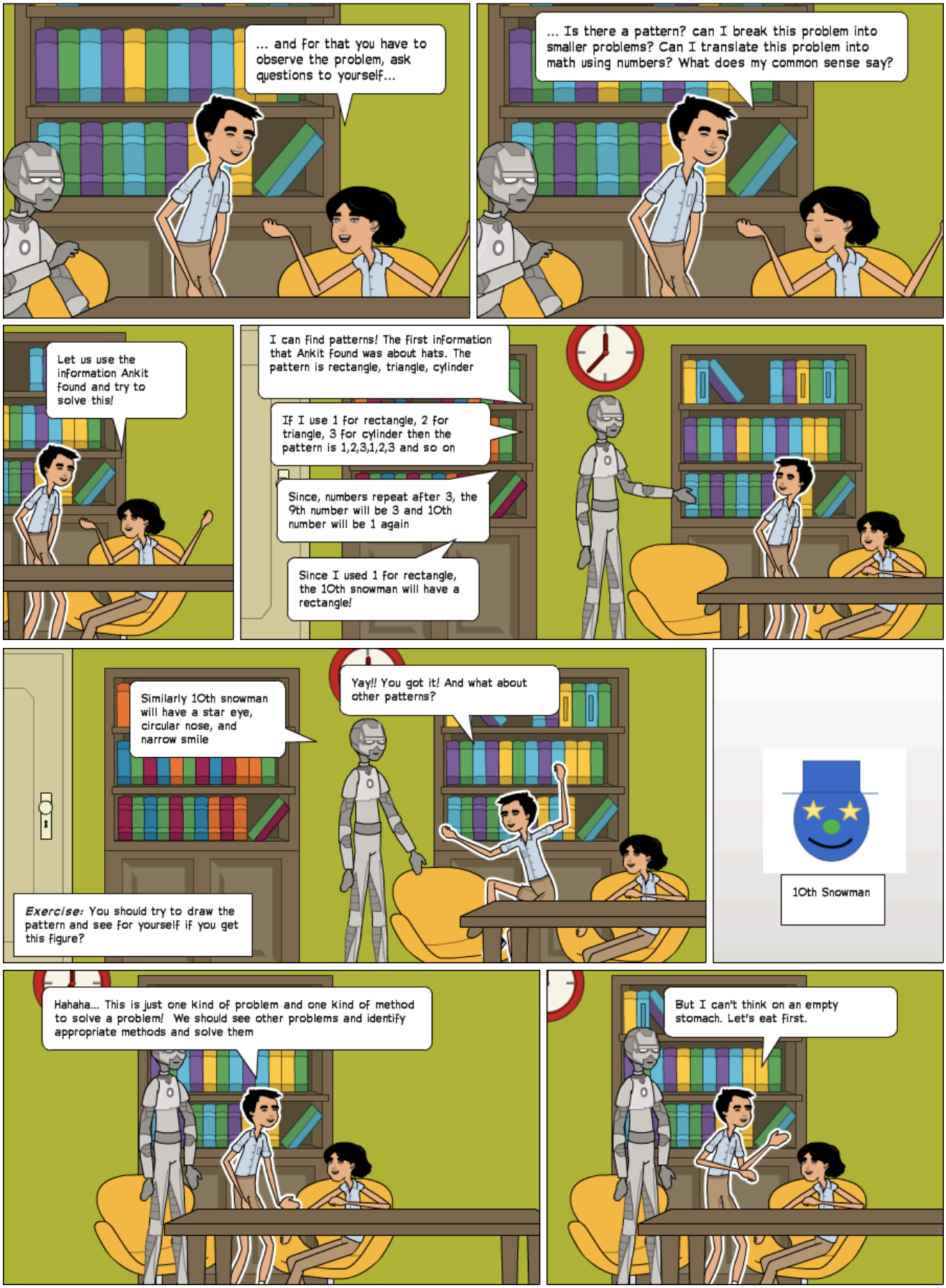











... and for that you have to observe the problem, ask questions to yourself...
... Is there a pattern? can I break this problem into smaller problems? Can I translate this problem into math using numbers? What does my common sense say?
Let us use the information Ankit found and try to solve this!
I can find patterns! The first information that Ankit found was about hats. The pattern is rectangle, triangle, cylinder
If I use 1 for rectangle, 2 for triangle, 3 for cylinder then the pattern is 1,2,3,1,2,3 and so on
Since, numbers repeat after 3, the 9th number will be 3 and 10th number will be 1 again
Since I used 1 for rectangle, the 10th snowman will have a rectangle!
Similarly 10th snowman will have a star eye, circular nose, and narrow smile
Yay!! You got it! And what about other patterns?
Exercise: You should try to draw the pattern and see for yourself if you get this figure?
10th Snowman
Hahaha... This is just one kind of problem and one kind of method to solve a problem! We should see other problems and identify appropriate methods and solve them
But I can't think on an empty stomach. Let's eat first.

Polya's Problem Solving framework

George Pólya was a Hungarian mathematician. He made fundamental contributions to combinatorics, number theory, numerical analysis and probability theory. He is also noted for his work in heuristics and mathematics education.

(December 13, 1887 – July 9th, 1985)

Questions you should ask to yourself

First step
You have to understand the problem

What is the problem? What is the unknown in the problem? What is the data and facts that we already know? What is the condition?

Second step
Find the connection between data and the unknown

Have you seen it before or in a slightly different form? Do you know a related problem? Look at the unknown and try to think of a familiar problem having the same or similar unknown.

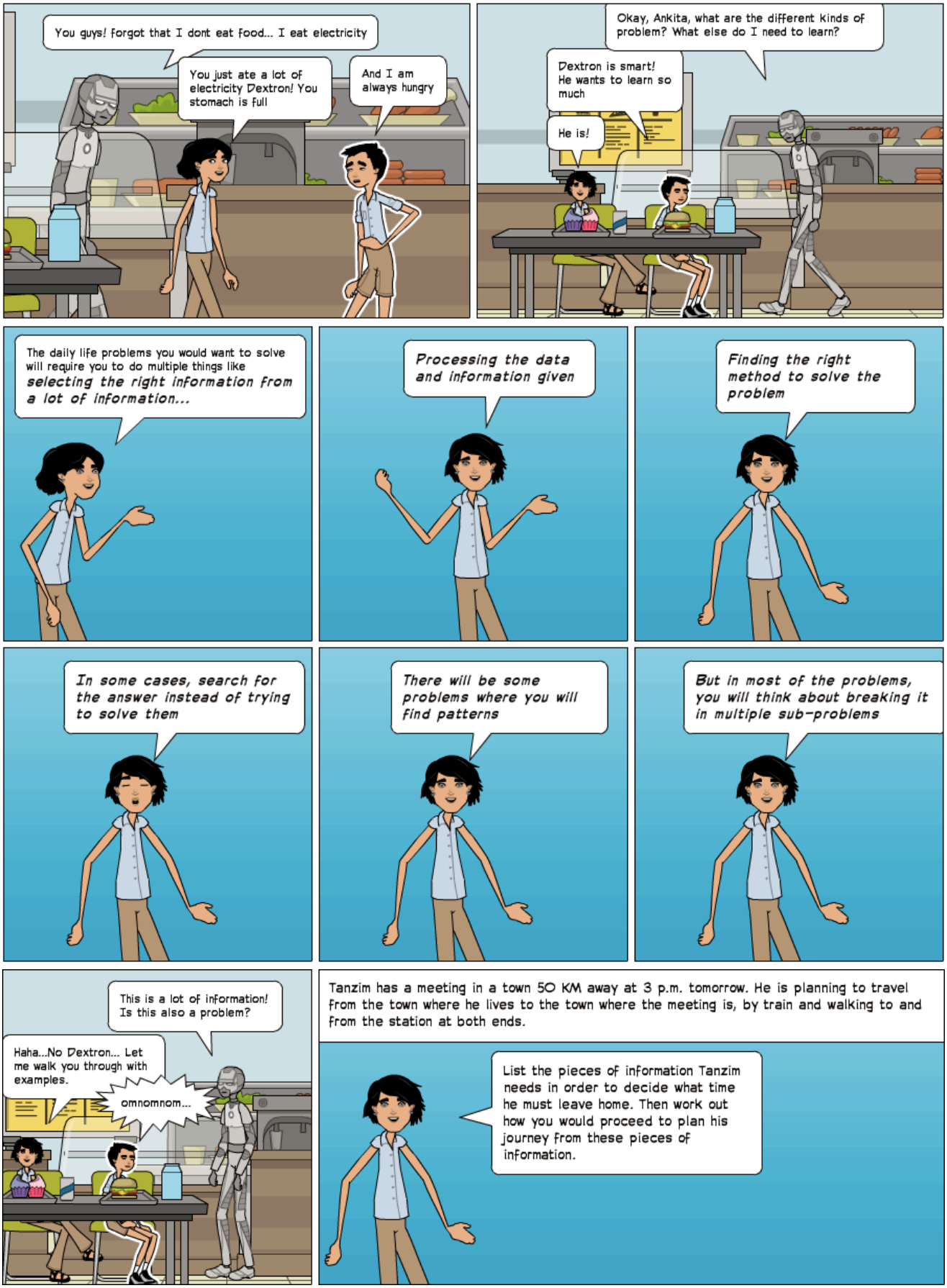
Third step
Carry out your plan

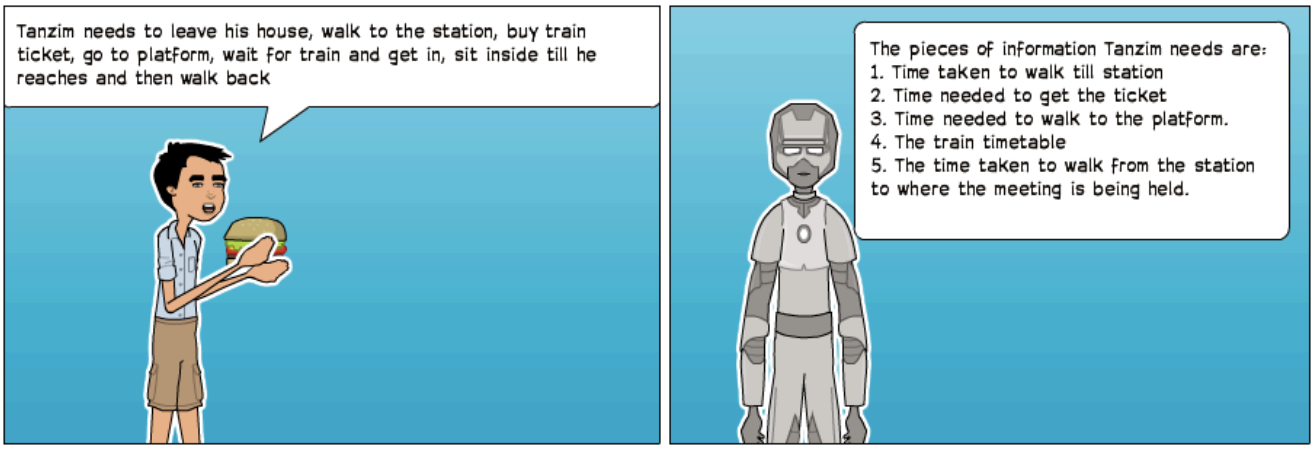
Carrying out a plan of the solution, check each step. Can you see clearly that the step is correct? Can you prove that it is correct?

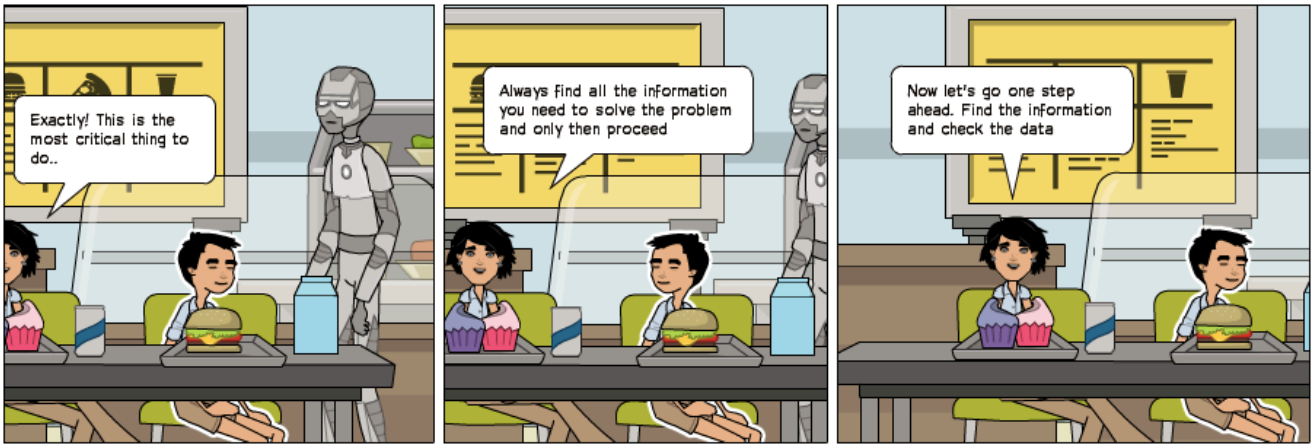
Fourth step
Examine the solution obtained

Can you check the result? Can you check the argument? Can you derive the results differently? Can you see the result, or method for some other problem?

I need this, I need this not


You guys! forgot that I dont eat food... I eat electricity
You just ate a lot of electricity Dextron! You stomach is full
And I am always hungry
Okay, Ankita, what are the different kinds of problem? What else do I need to learn?
Dextron is smart! He wants to learn so much
He is!
The daily life problems you would want to solve will require you to do multiple things like selecting the right information from a lot of information...
Processing the data and information given
Finding the right method to solve the problem
In some cases, search for the answer instead of trying to solve them
There will be some problems where you will find patterns
But in most of the problems, you will think about breaking it in multiple sub-problems
This is a lot of information! Is this also a problem?
Haha...No Dextron... Let me walk you through with examples.
omnomnom...
Tanzim has a meeting in a town 50 KM away at 3 p.m. tomorrow. He is planning to travel from the town where he lives to the town where the meeting is, by train and walking to and from the station at both ends.
List the pieces of information Tanzim needs in order to decide what time he must leave home. Then work out how you would proceed to plan his journey from these pieces of information.

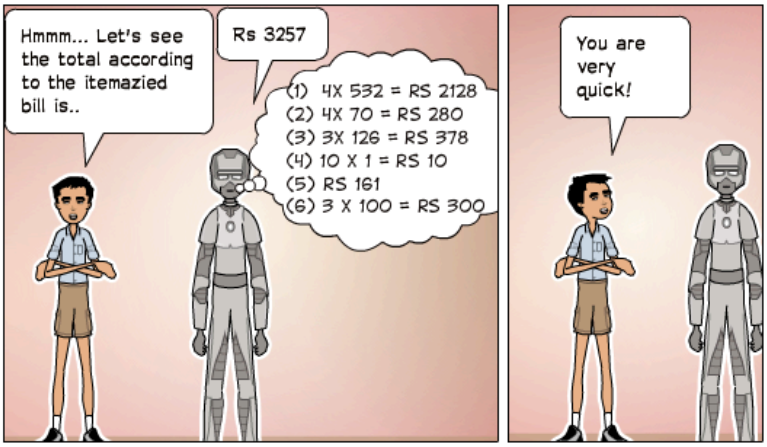


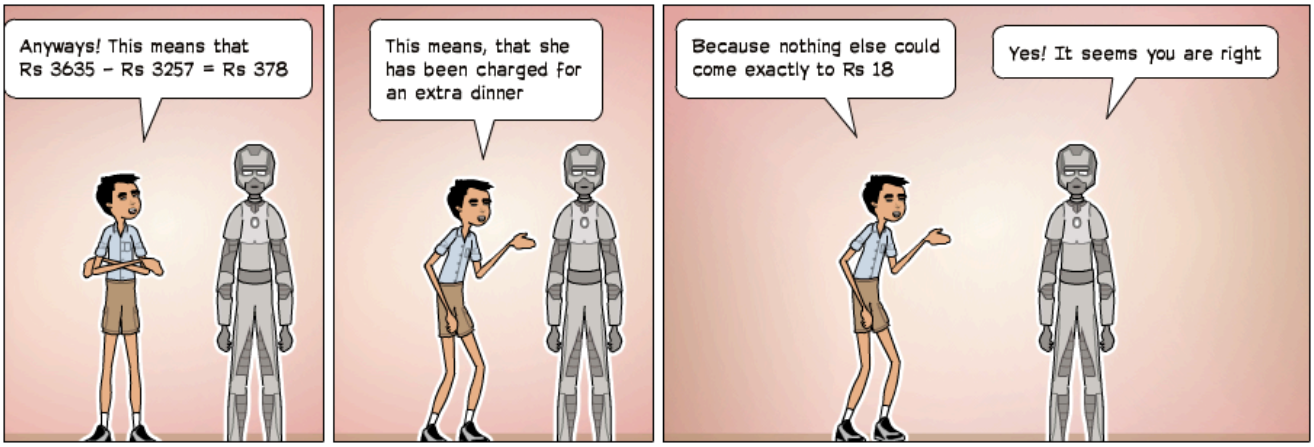


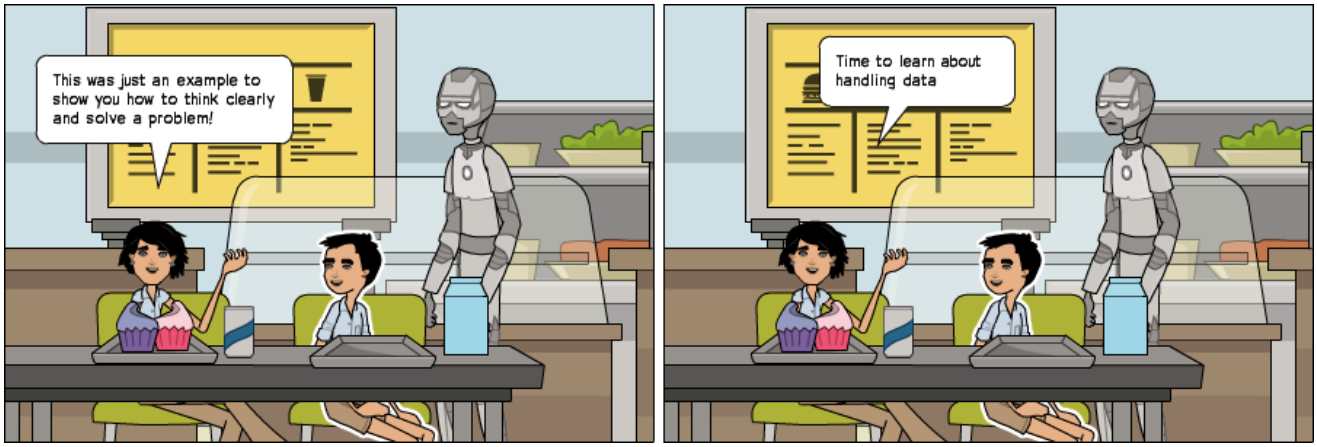
RHEA HAS BEEN STAYING IN A HOTEL ON A BUSINESS TRIP. WHEN SHE CHECKS OUT, THE HOTEL'S COMPUTER ISN'T WORKING, SO THE RECEPTIONIST MAKES A BILL BY HAND FROM THE RECEIPTS, TOTALING ***RS. 3635***. RHEA THINKS SHE HAS BEEN OVERCHARGED, SO SHE CHECKS THE ITEMIZED BILL CAREFULLY.

ROOM: 4 NIGHTS AT RS. 532.00 PER NIGHT
BREAKFAST: 4 AT RS. 70.00 EACH
DINNERS: 3 AT RS. 126.00 EACH
TELEPHONE: 10 UNITS AT RS 1 PER UNIT
RESTAURANT: VARIOUS DRINKS TOTALING RS. 161.00
LAUNDRY: 3 DRESSES AT RS. 100.00 EACH

IT APPEARS THAT THE RECEPTIONIST MISCOUNTED ONE OF THE ITEMS WHEN ADDING UP THE TOTAL. WHICH ITEM HAS RHEA BEEN CHARGED TOO MUCH FOR?







Practice Problem

Problem 1

Rajesh is cooking a meal for some friends.

This will involve roasting corns (makka), which will take 2 hours' cooking time plus 15 minutes resting on removal from the oven. The oven takes 15 minutes to warm up. He will also cook some rice (30 minutes' soaking plus 15 minutes' cooking), cabbage (5 minutes to prepare and 5 minutes to cook) and a tomato sauce (10 minutes to prepare and 15 minutes to cook).

What should be the timing of each step of cooking the meal if the friends are to eat at 7 p.m.?

Problem 2

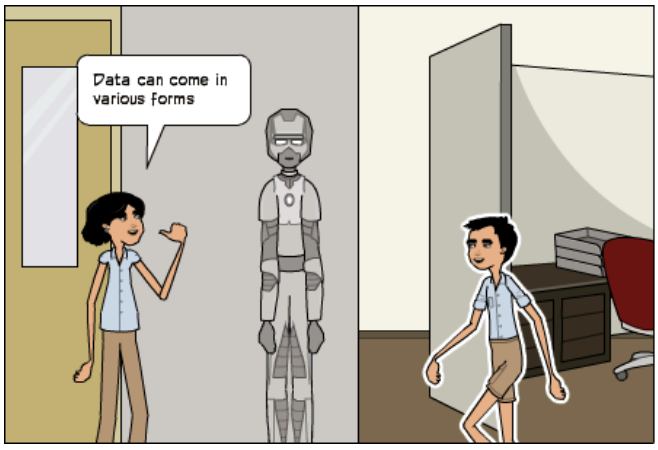
The SuperSave supermarket sells Birma washing powder for Rs 1.20 a bottle. At this price they are charging 50% more than the price at which they buy the item from the manufacturers. Next week SuperSave is having a 'Buy two get a third free' offer on this item. The supermarket does not want to lose money on this offer, so it expects the manufacturers to reduce their prices so SuperSave will make the same actual profit on every three bottles sold.

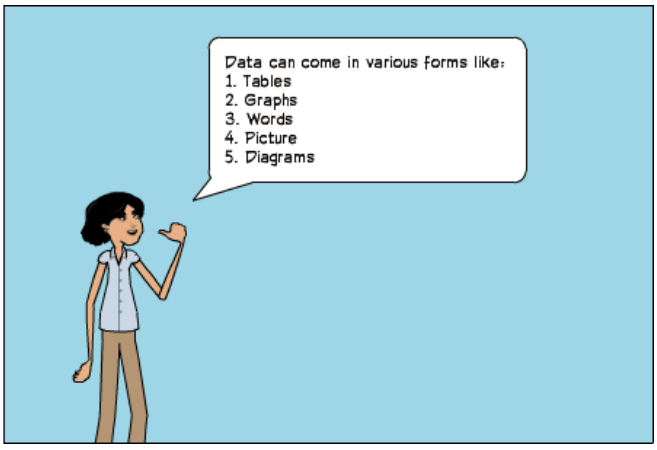
By how much will the manufacturers have to reduce their prices?

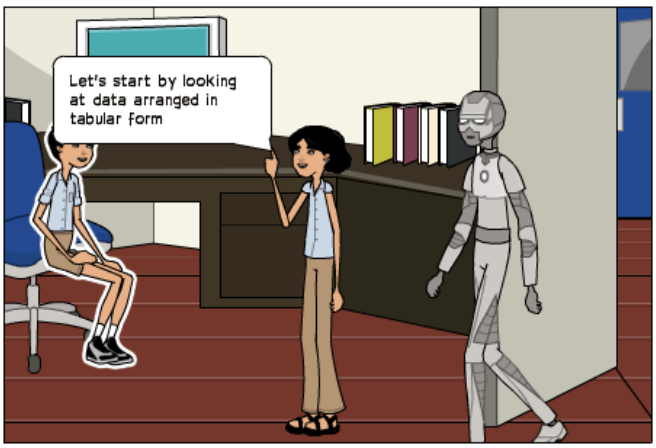
(a) 1/6
(b) 1/4
(c) 1/3
(d) 1/2
(e) 2/3

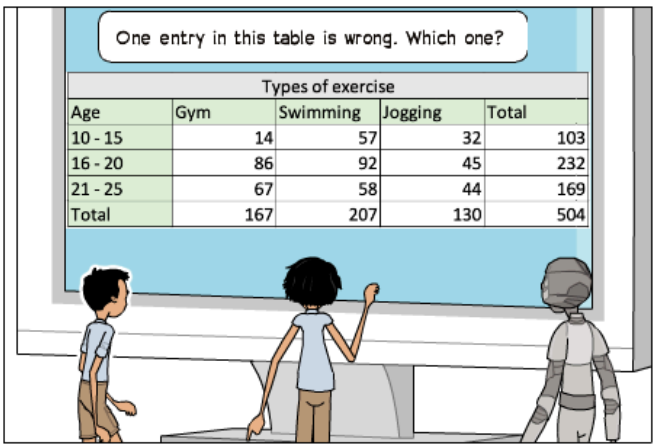
Answer:
Problem 1 – Put all the steps for cooking in a table and start backwards. The first steps will be to turn the oven on at 4:30 PM.
Problem 2 – Option D

Itni shakti humein dena data

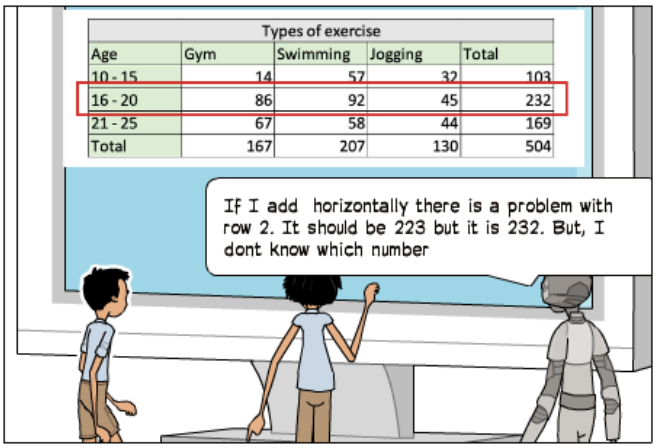








Types of exercise				
Age	Gym	Swimming	Jogging	Total
10 - 15	14	57	32	103
16 - 20	86	92	45	232
21 - 25	67	58	44	169
Total	167	207	130	504



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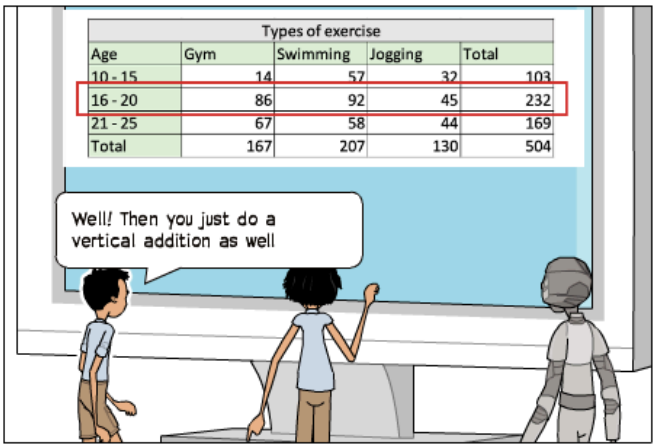

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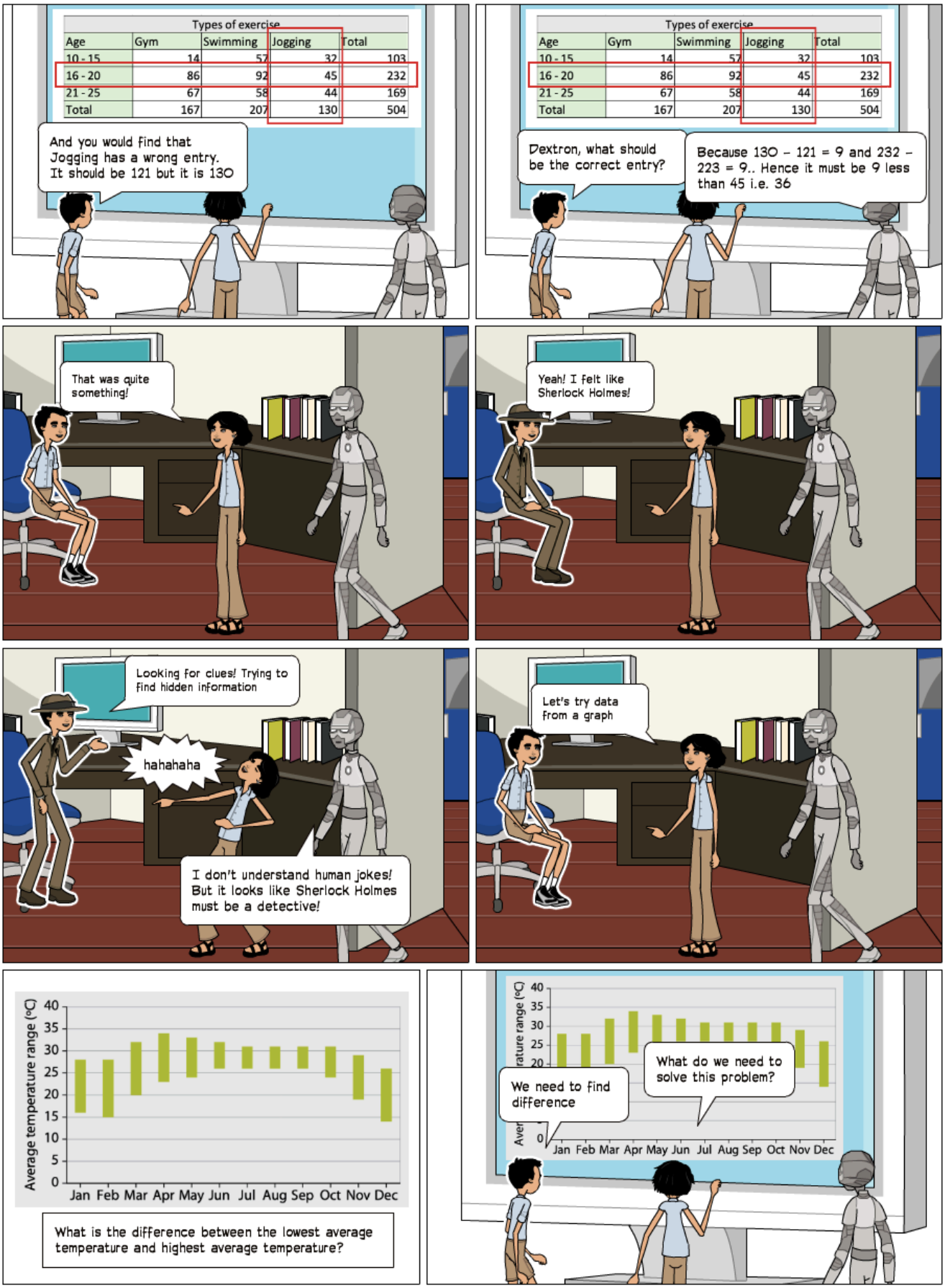
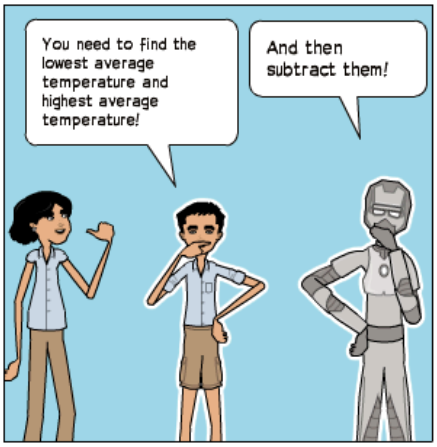
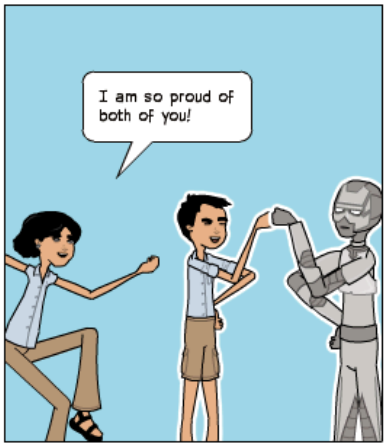
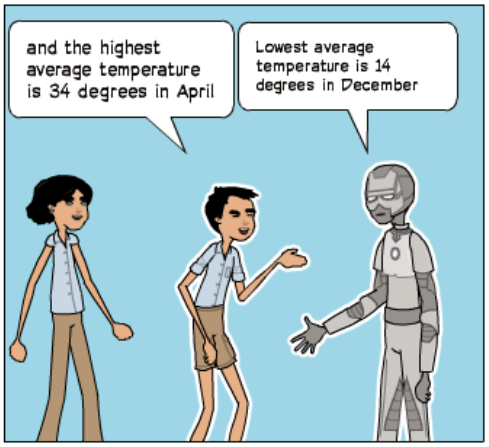
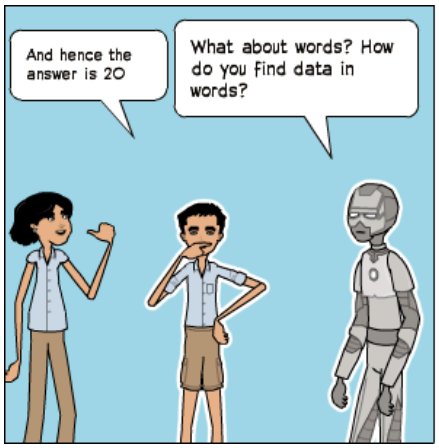
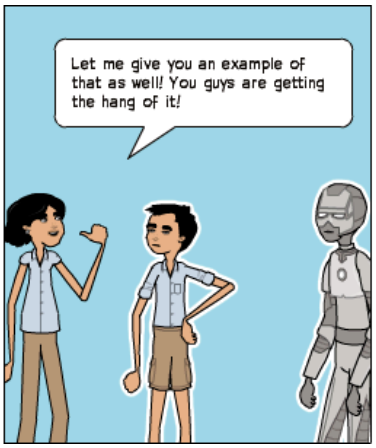

Types of exercise
Age | Gym | Swimming | Jogging | Total
10 - 15 | 14 | 57 | 32 | 103
16 - 20 | 86 | 92 | 45 | 232
21 - 25 | 67 | 58 | 44 | 169
Total | 167 | 207 | 130 | 504
And you would find that Jogging has a wrong entry. It should be 121 but it is 130
Dextron, what should be the correct entry?
Because 130 – 121 = 9 and 232 – 223 = 9.. Hence it must be 9 less than 45 i.e. 36
That was quite something!
Yeah! I felt like Sherlock Holmes!
Looking for clues! Trying to find hidden information
hahahaha
I don't understand human jokes! But it looks like Sherlock Holmes must be a detective!
Let's try data from a graph
Average temperature range (°C)
0 5 10 15 20 25 30 35 40
Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec
What is the difference between the lowest average temperature and highest average temperature?
We need to find difference
What do we need to solve this problem?







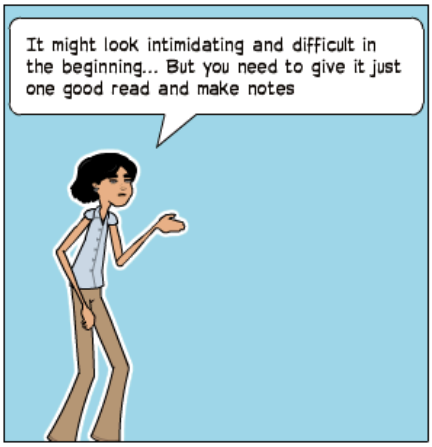


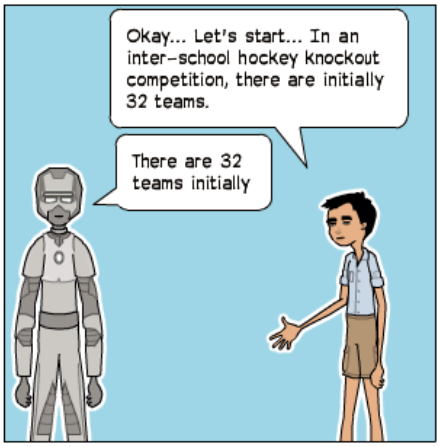


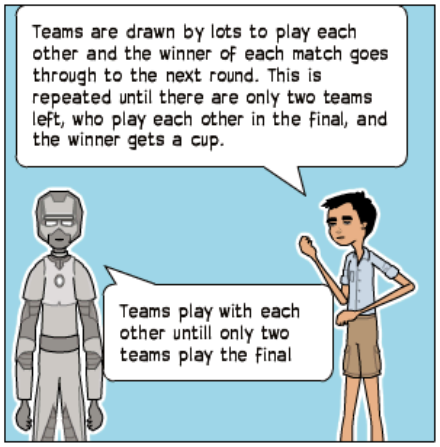
In an inter-school hockey knockout competition, there are initially 32 teams. Teams are drawn by lots to play each other and the winner of each match goes through to the next round. This is repeated until there are only two teams left, who play each other in the final, and the winner gets a cup. Matches have two halves of 20 minutes each. If the teams are level at the end of normal play, two extra 10-minute periods are played. If it is still a draw, teams take penalty shots at goal to decide the winner.

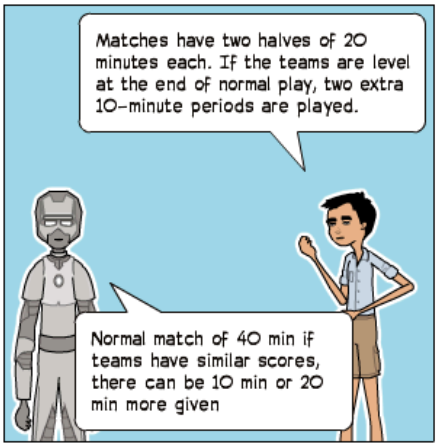
High-school was eventually knocked out in the semi-final (without extra time). In one of the earlier rounds they had to play the two extra periods before they won.

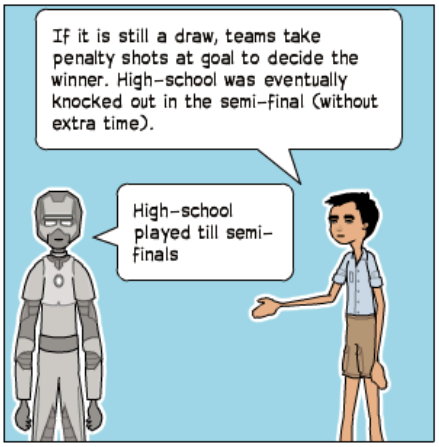
For how long in total had High-school played when they were knocked out?

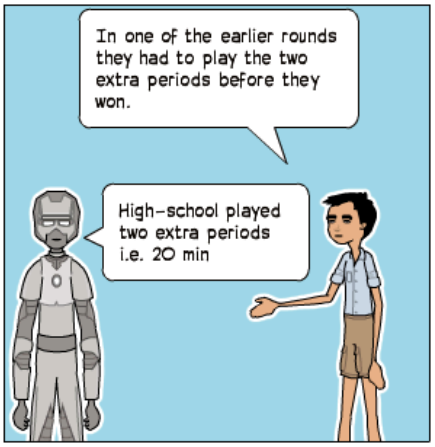


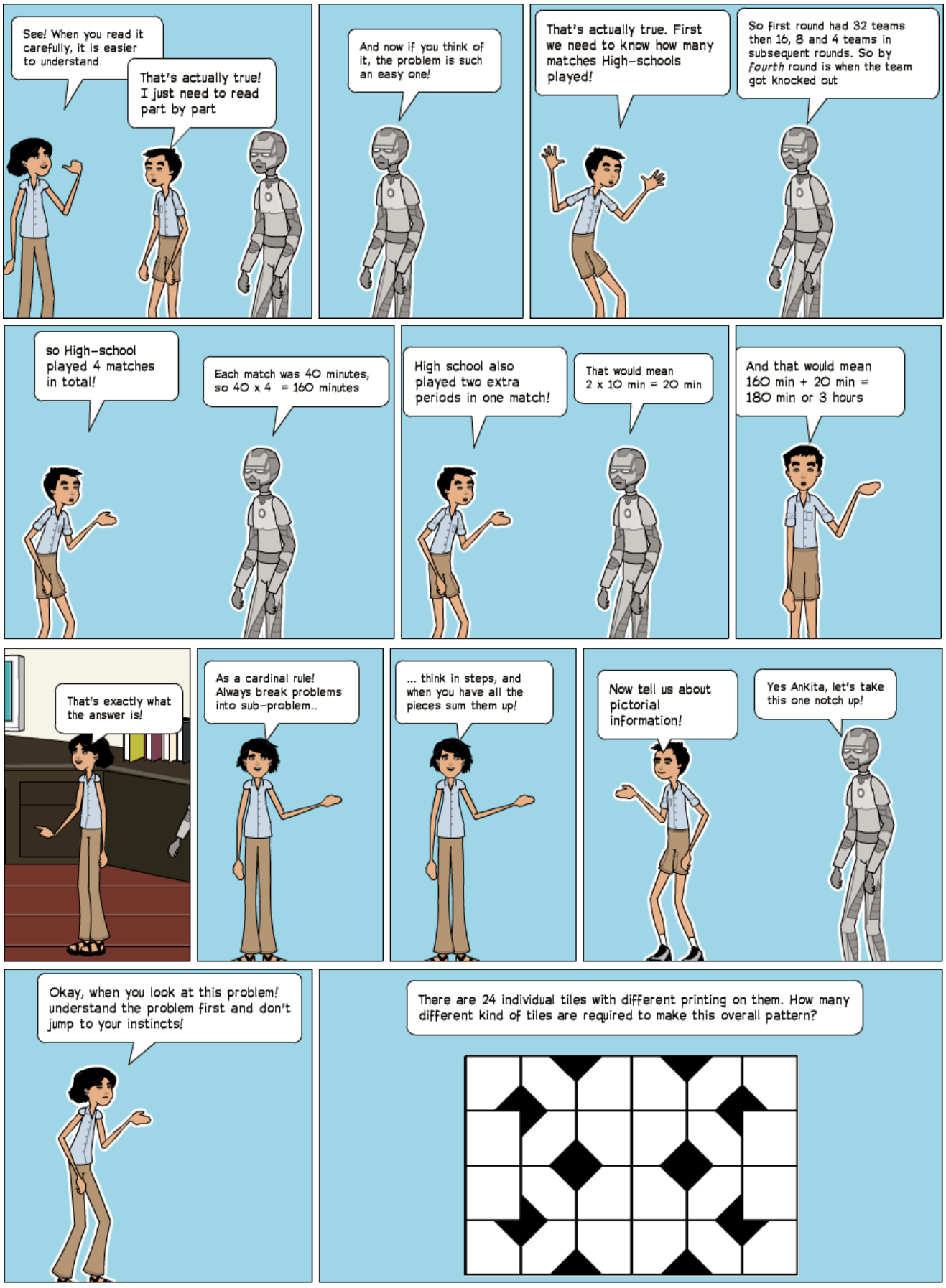


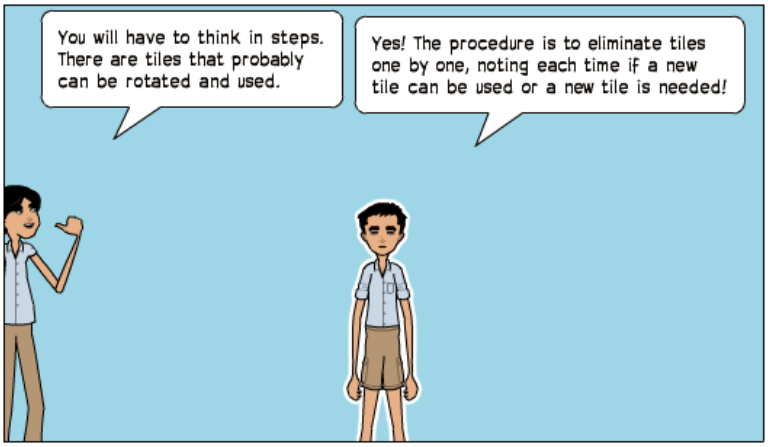


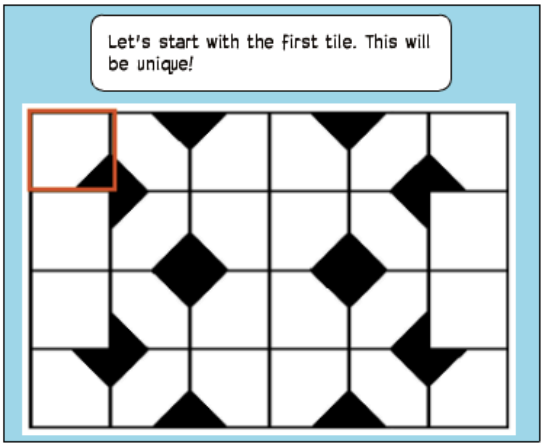


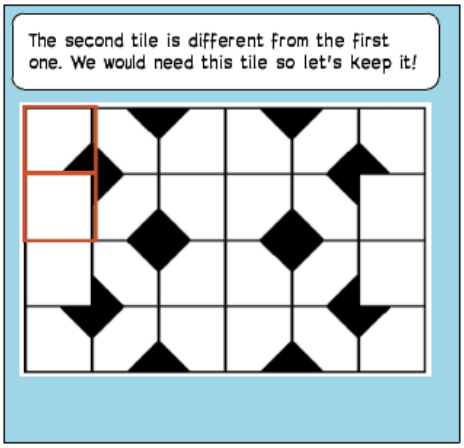


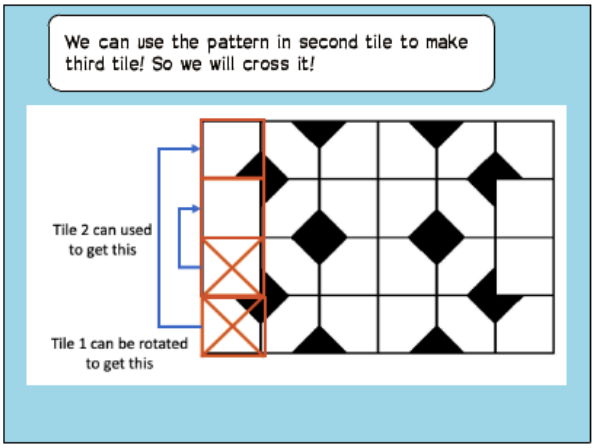


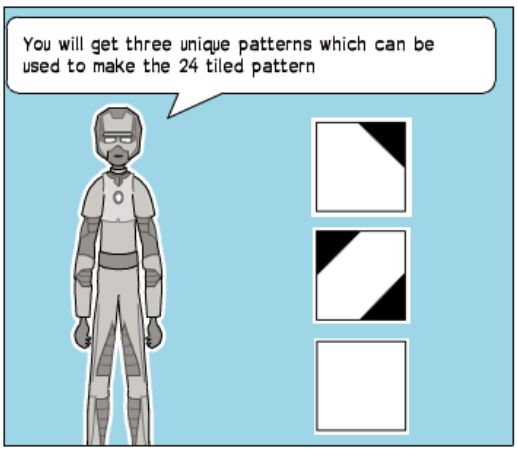

See! When you read it carefully, it is easier to understand
That's actually true! I just need to read part by part
And now if you think of it, the problem is such an easy one!
That's actually true. First we need to know how many matches High-schools played!
So first round had 32 teams then 16, 8 and 4 teams in subsequent rounds. So by *Fourth* round is when the team got knocked out
so High-school played 4 matches in total!
Each match was 40 minutes, so 40 x 4 = 160 minutes
High school also played two extra periods in one match!
That would mean 2 x 10 min = 20 min
And that would mean 160 min + 20 min = 180 min or 3 hours
That's exactly what the answer is!
As a cardinal rule! Always break problems into sub-problem..
... think in steps, and when you have all the pieces sum them up!
Now tell us about pictorial information!
Yes Ankita, let's take this one notch up!
Okay, when you look at this problem! understand the problem first and don't jump to your instincts!
There are 24 individual tiles with different printing on them. How many different kind of tiles are required to make this overall pattern?

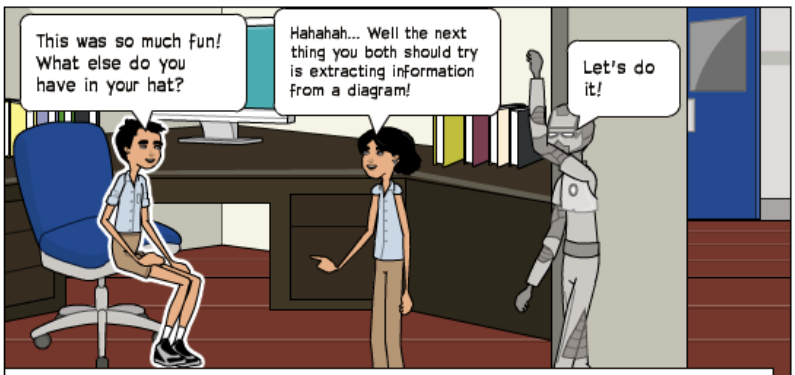






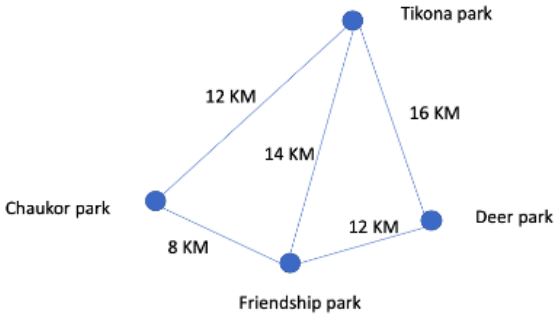


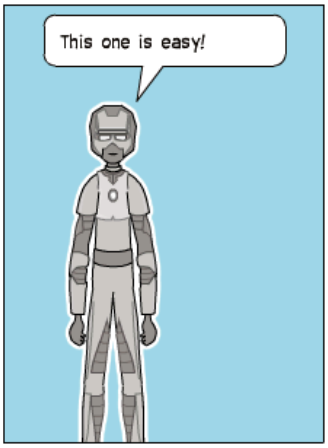


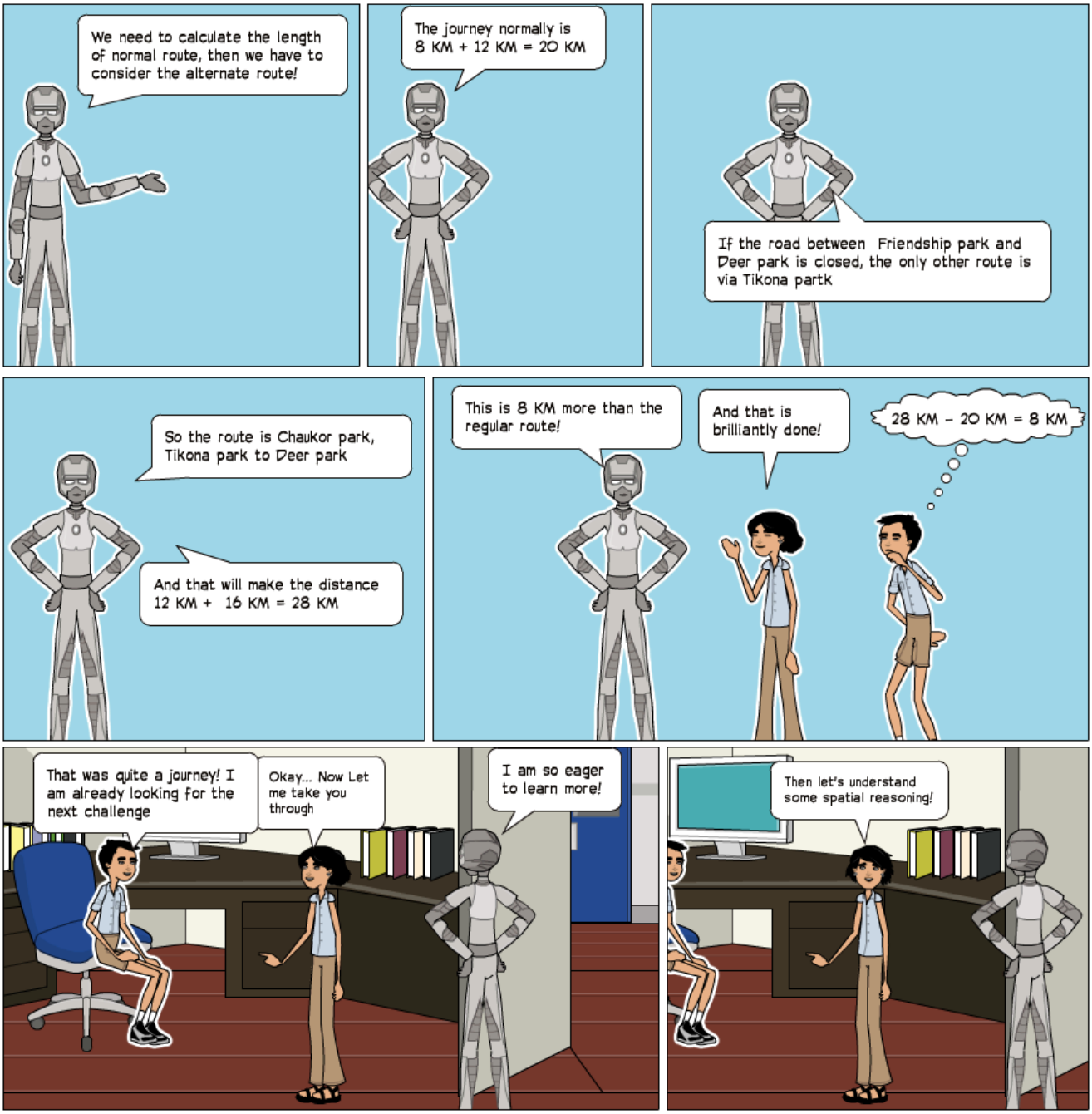


What else do you have in your hat is a phrase used to say what is the next thing ou will bring up

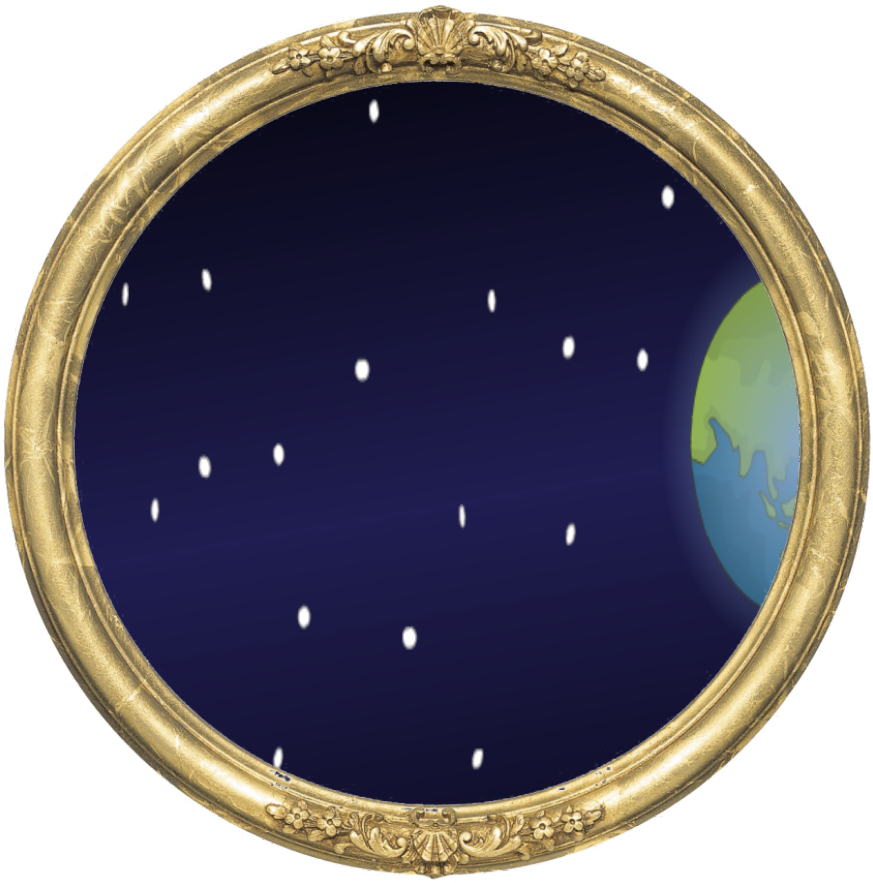
I live in Chaukor park and wish to visit a friend in Deer park. I normally go via Friendship park but have discovered (before setting off) that the road between Friendship park and Deer park is blocked by an accident.
How much will this add to my journey?

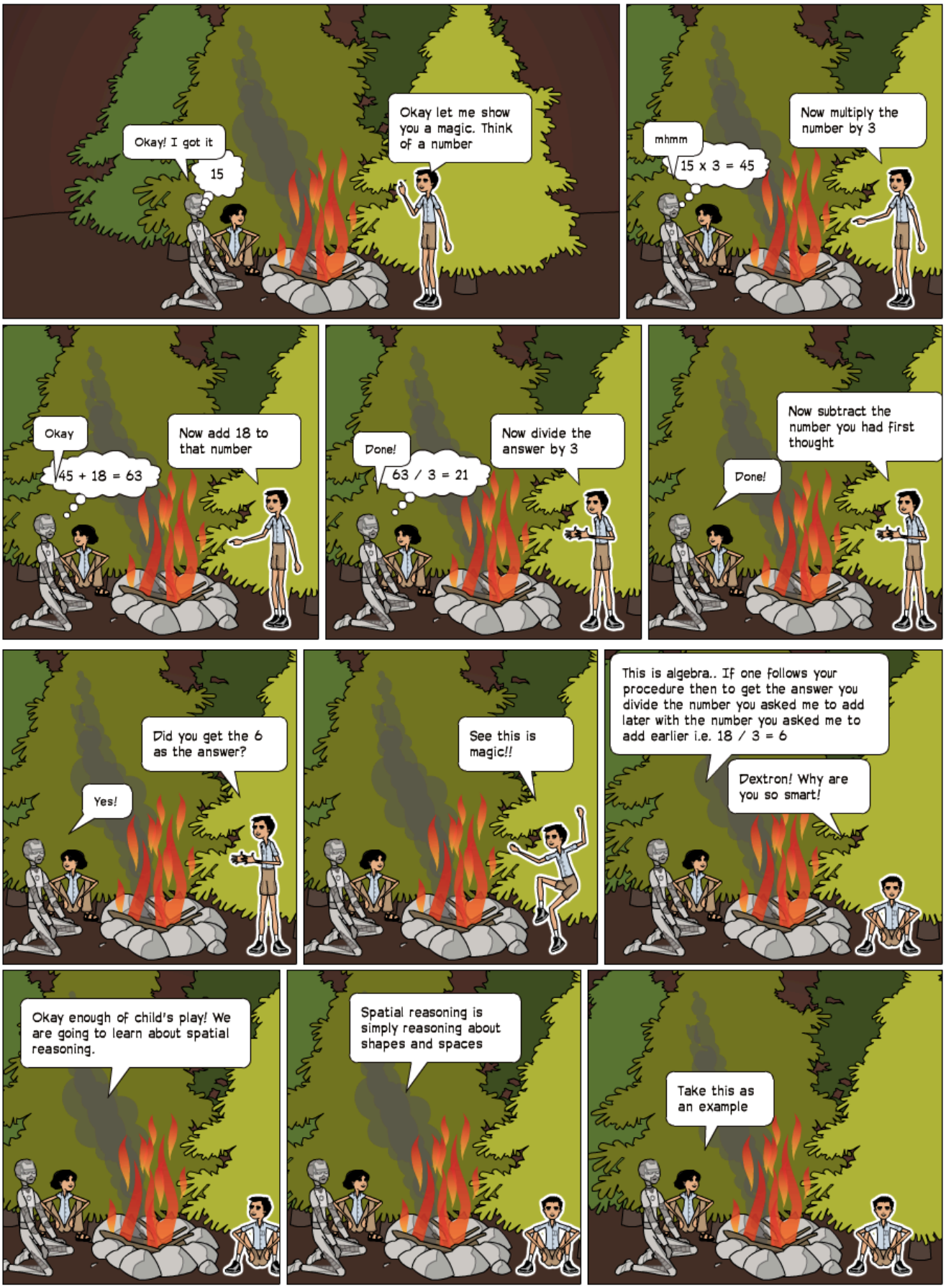




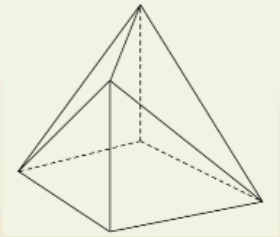

We need to calculate the length of normal route, then we have to consider the alternate route!
The journey normally is 8 KM + 12 KM = 20 KM
If the road between Friendship park and Deer park is closed, the only other route is via Tikona partk
So the route is Chaukor park, Tikona park to Deer park
And that will make the distance 12 KM + 16 KM = 28 KM
This is 8 KM more than the regular route!
And that is brilliantly done!
28 KM – 20 KM = 8 KM
That was quite a journey! I am already looking for the next challenge
Okay... Now Let me take you through
I am so eager to learn more!
Then let's understand some spatial reasoning!

Give me some space!

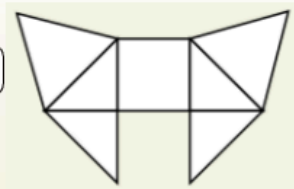



Okay! I got it
15
Okay let me show you a magic. Think of a number
mhmm
15 x 3 = 45
Now multiply the number by 3
Okay
45 + 18 = 63
Now add 18 to that number
Done!
63 / 3 = 21
Now divide the answer by 3
Done!
Now subtract the number you had first thought
Yes!
Did you get the 6 as the answer?
See this is magic!!
This is algebra.. If one follows your procedure then to get the answer you divide the number you asked me to add later with the number you asked me to add earlier i.e. 18 / 3 = 6
Dextron! Why are you so smart!
Okay enough of child's play! We are going to learn about spatial reasoning.
Spatial reasoning is simply reasoning about shapes and spaces
Take this as an example

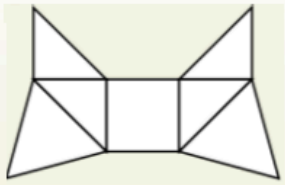
This is a 3 dimensional shape! Which of the following pieces of cardboard shape will fold to make the shape?



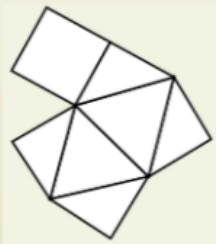
A



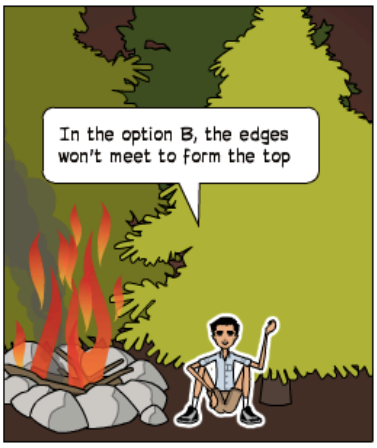
C

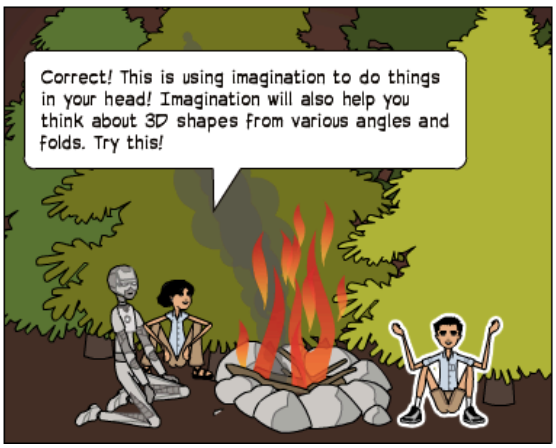


B

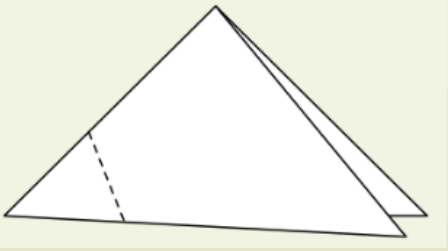


D

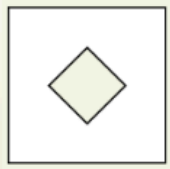




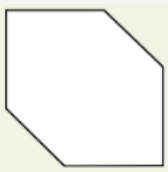
Some children are making decorations. A square sheet of paper is folded along a diagonal and then again so the two sharp points meet, as shown. A cut is made through all the layers of paper along the dotted line shown and the small pieces removed. The paper is then opened. What does it look like?



A



B



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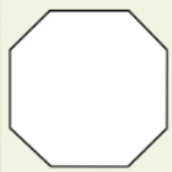
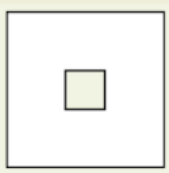
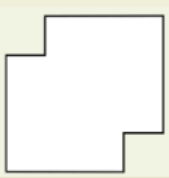
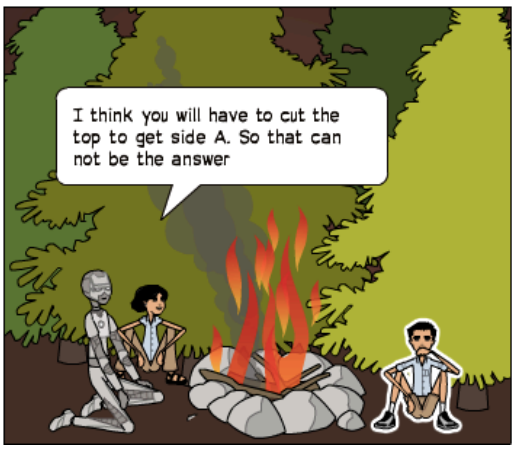


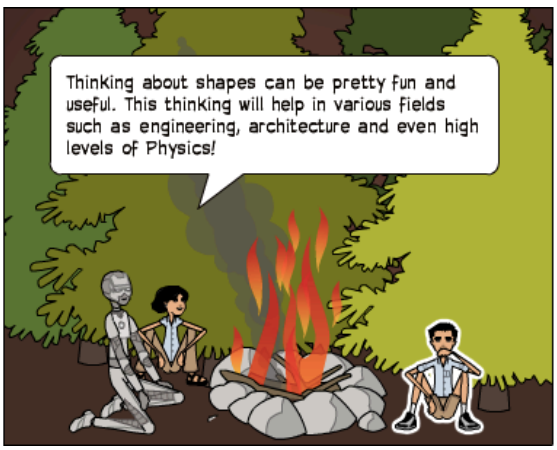
D



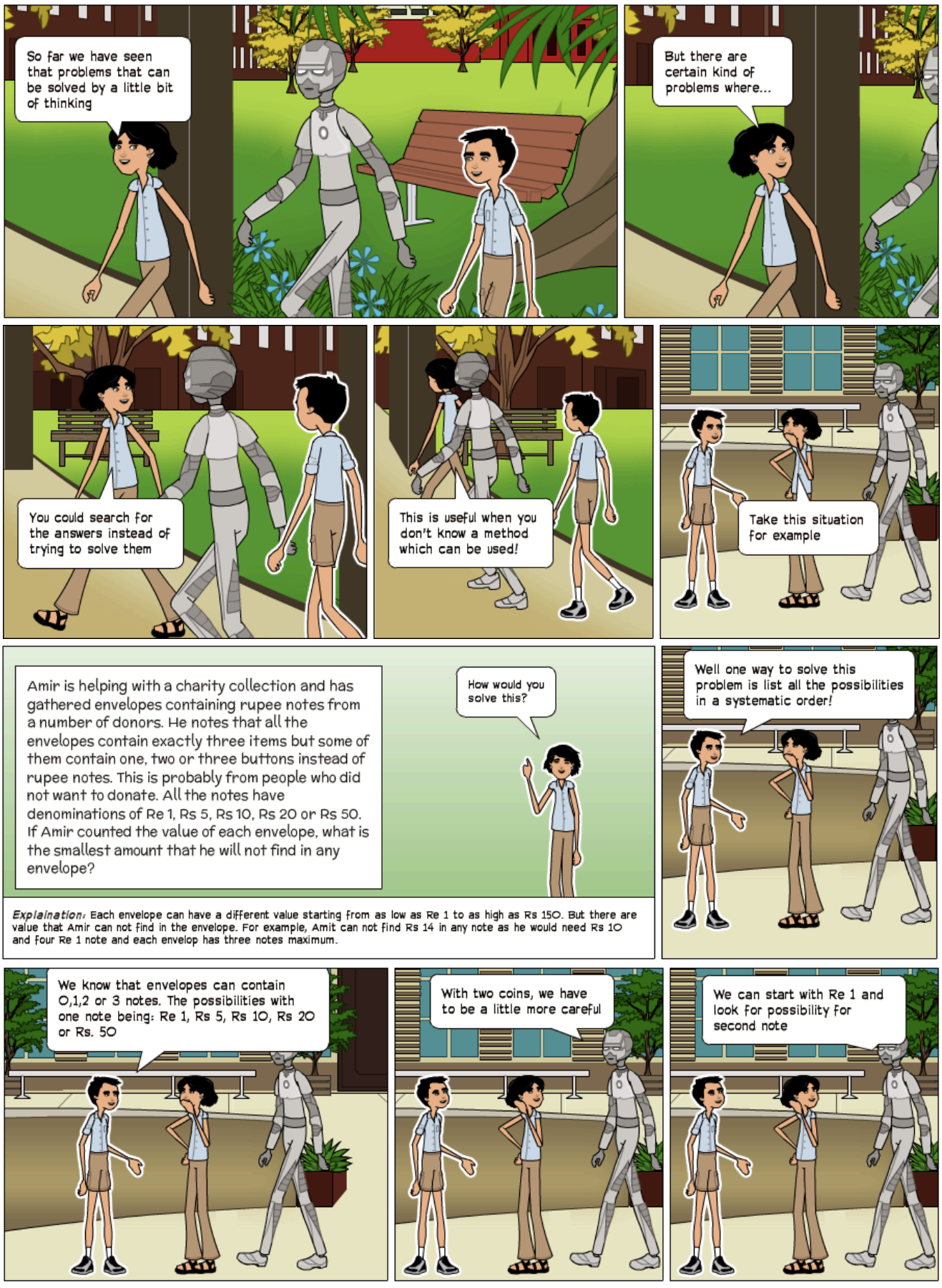
E

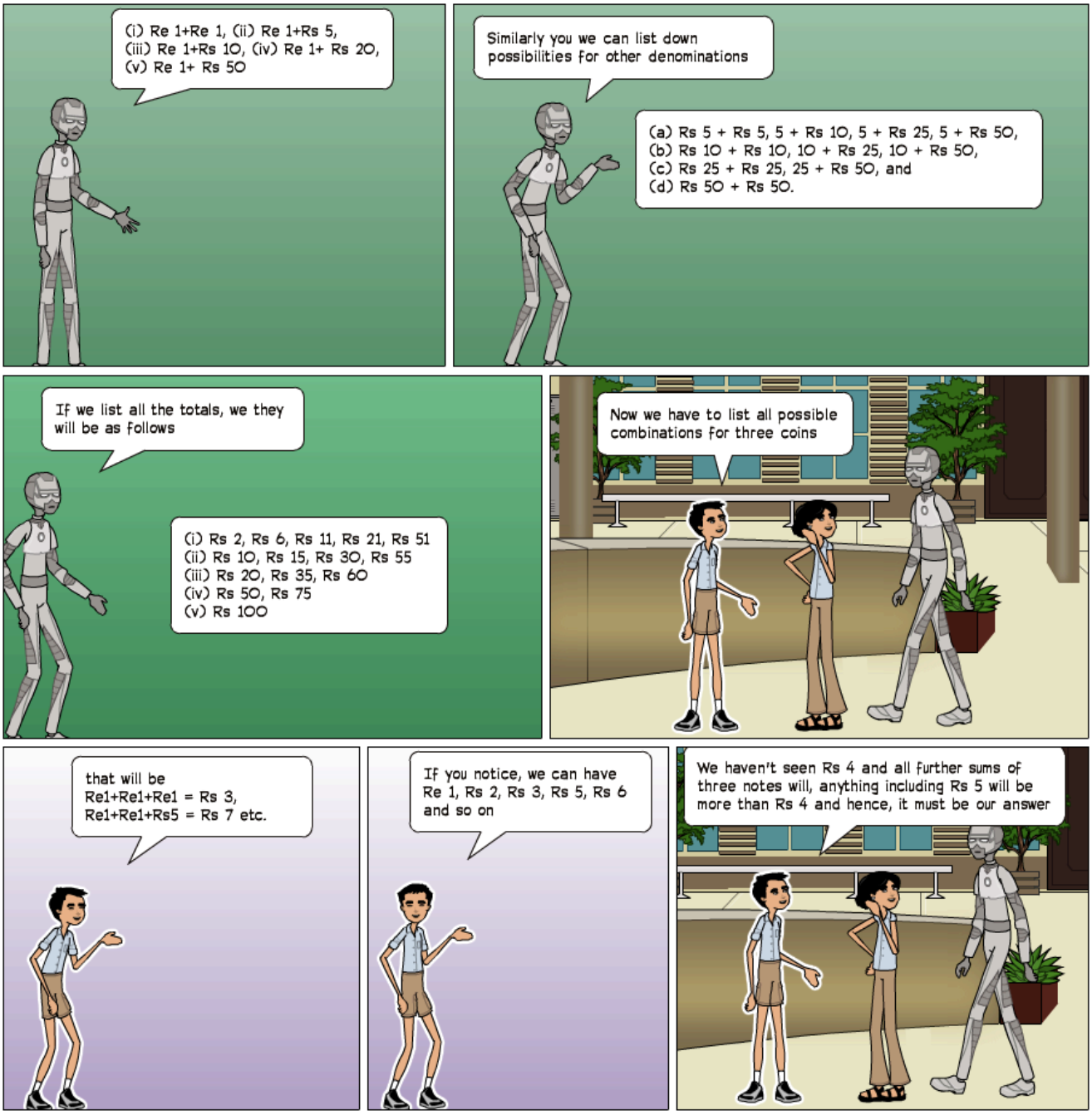






If you can't solve it, search for it !


So far we have seen that problems that can be solved by a little bit of thinking
But there are certain kind of problems where...
You could search for the answers instead of trying to solve them
This is useful when you don't know a method which can be used!
Take this situation for example
Amir is helping with a charity collection and has gathered envelopes containing rupee notes from a number of donors. He notes that all the envelopes contain exactly three items but some of them contain one, two or three buttons instead of rupee notes. This is probably from people who did not want to donate. All the notes have denominations of Re 1, Rs 5, Rs 10, Rs 20 or Rs 50. If Amir counted the value of each envelope, what is the smallest amount that he will not find in any envelope?
How would you solve this?
Well one way to solve this problem is list all the possibilities in a systematic order!
Explaination: Each envelope can have a different value starting from as low as Re 1 to as high as Rs 150. But there are value that Amir can not find in the envelope. For example, Amit can not find Rs 14 in any note as he would need Rs 10 and four Re 1 note and each envelop has three notes maximum.
We know that envelopes can contain 0,1,2 or 3 notes. The possibilities with one note being: Re 1, Rs 5, Rs 10, Rs 20 or Rs. 50
With two coins, we have to be a little more careful
We can start with Re 1 and look for possibility for second note



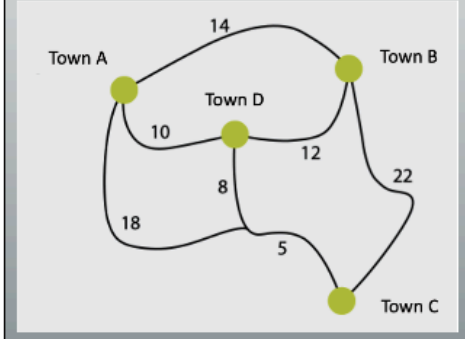
If you list down all the values that envelop can have:

Re 1, Rs 2, Rs 3, Rs 4, Rs 5, Rs 6, Rs 7, Rs 8, Rs 9 and so on

Then you can start striking off the numbers that you have found as the problem asks you to find the number that is not possible to get in the envelope. In this case:

~~Re 1~~, ~~Rs 2~~, ~~Rs 3~~, Rs 4, ~~Rs 5~~, ~~Rs 6~~, ~~Rs 7~~, ~~Rs 8~~, ~~Rs 9~~ and so on

Take another example, where it is much easier if you try searching for the solution



I stay in Town B and have to deliver groceries in the other three town and finally return to Town B. What is the shortest distance I have to drive?

In such problems, the best way to solve is list all the paths

Since there are very limited number of paths we should list all of them down along with the total distance

These are all the possible paths

BADCB = 14+10+8+5+22 = 59

BACDB = 14+18+5+5+8+12 = 62

BDACB = 12+10+18+5+22 = 67

BDCAB = 12+8+5+5+18+14 = 62

BCDAB = 22+5+8+10+14 = 59

BCADB = 22+5+18+10+12 = 67

Once we have listed all these possibilities, it is easy to find the path. In this case, there are two paths, both of them will take 59 KM

This strategy is useful when you can not find or remember a conventional method to solve the problem

Ankita, I have a question! Can we solve all the problems that exist? I mean are there any problems that can not be solved?

That's a very good question! There are certain types of problems which have not been solved yet, but most of the problems we deal with in day-to-day life can be solved through a little bit of data and thinking.

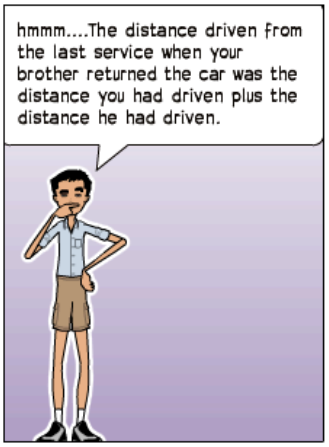
Take this for example

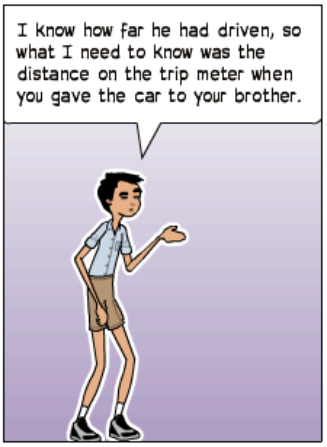
I use the trip meter on my car to measure the distance driven since I last had the car serviced, so that I know when the next service is due. The trip meter can be set to zero by the press of a button and records the kilometres driven since it was last reset.

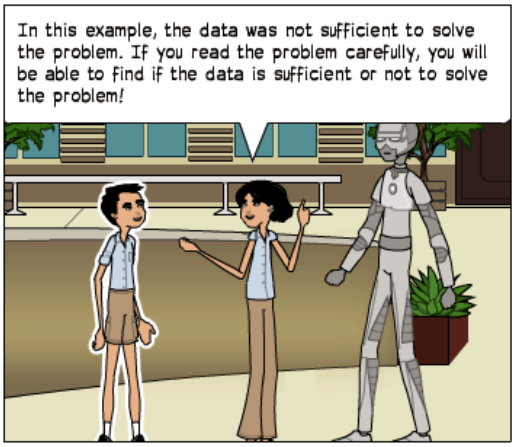
I set the trip meter to zero after my last service. The next service is due after 20,000 km have been driven. Some time later, I lent the car to my brother. I forgot to tell him about the trip meter; he pressed the button to zero it and drove 575 km. I then started driving again without realising what he had done.

What should the trip meter read when the next service is due?

The above problem cannot be solved with the information given. What additional piece of information is needed to solve it?







I have a small collection of three different types of old coin (1 paisa coins, 2 paisa coins and 5 paisa coins). The collection contains 15 coins in total. There are more 1 Paisa coin than 2 Paisa coins and more 2 Paisa coins than 5 Paisa coins. Which one of the following single pieces of information would enable you to know exactly how many of each type of coin there was?

(a) There are four more 2 Paisa coin than 5 Paisa coins

(b) There are five more 1 paisa coin than 5 paisa coins

(c) There are three more 1 paisa coin than 2 paisa coins.

(d) There is one less 1 paisa coin than 5 paisa coins and 2 paisa coins together.

1 paisa coin	2 paisa coin	5 paisa coin
12	2	1

Answer: Eight 1 Paisa coins, Five 2 Paisa coins, Two 5 paisa coins

Acknowledgement

We acknowledge the work done by Mr. John Butterworth and Mr. Geoff Thwaites on critical thinking through their book – Thinking Skills (Critical Thinking and Problem Solving)
This book has been a big source of inspiration for this book

Additional resources

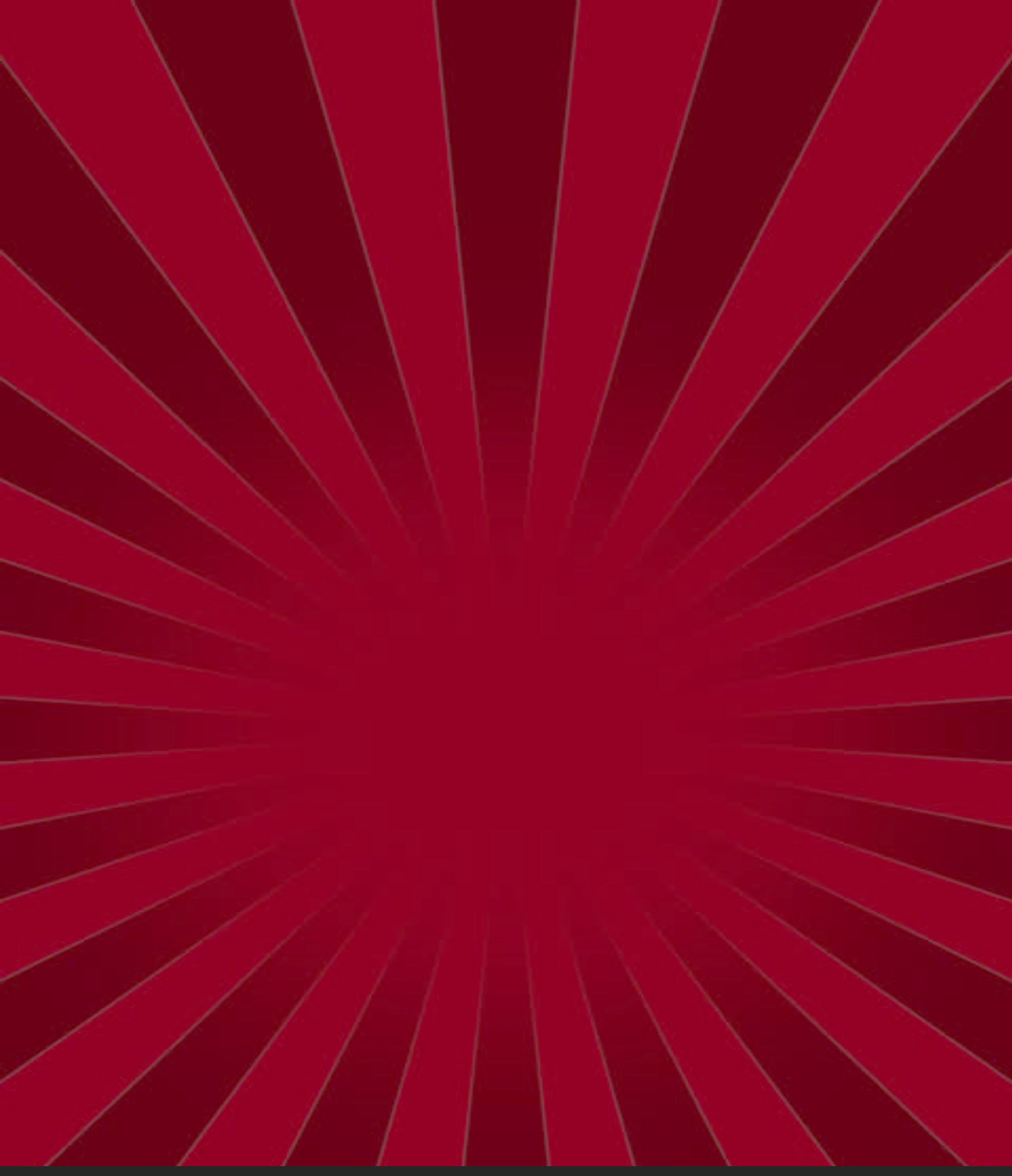
Readings

1. Hands on Maths
2. Mathematics Can Be Fun
3. Great Ideas of Modern Mathematics
4. Polya's Problem Solving strategy

Videos

1. TED-Ed - Riddles
2. TED-Ed – Think like a coder

We thank all the contributors for making the above resources available on the internet. We specially thank **Padma Shree Arvind Gupta** for making excellent learning resources available for free on the internet.



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